Topic Study Group 27

The Role of History of Mathematics in Education

Ysette Weiss[1] and Desiree Agterberg[2]

1. Rationale and Aim of the TSG

Mathematics is a human intellectual enterprise with a long history and a vivid present. Thus, mathematical knowledge is determined not only by the circumstances in which it becomes a deductively structured theory, but also by the procedure that originally led or may lead to it, and which is indispensable for understanding processes of change in mathematics. Therefore, learning mathematics includes not only the "polished products" of mathematical activity but also the understanding of (implicit) motivations, the sense-making actions, and the reflective processes of mathematicians, which aim to the construction of meaning. Hence, teaching mathematics should include giving the opportunity to students to "experience mathematics in the making." That is, although the "polished products" of mathematics form that part of mathematical knowledge that is communicated, criticized (in order to be accepted or rejected), and serving as the basis for new work, the process of producing mathematical knowledge is equally important, especially from a didactical point of view. This perception of mathematics should be central in the teaching of mathematics, and the image of mathematics communicated to the outside world. In this perspective, putting emphasis on integrating historical and epistemological issues in mathematics teaching and learning constitutes a possible natural way for exposing mathematics in the making that may lead to a better understanding of specific parts of mathematics and to a deeper awareness of what mathematics as a discipline is.

TSG-27 aims to provide a forum for participants to share their research interests and results, as well as their teaching ideas and classroom experience in connection with the integration of the history of mathematics in mathematics education. Special care is taken to present and promote ideas and research results of an as broad as possible international interest, while still focusing due attention to the national aspects of research and teaching experience in this area. Every effort will be made to allow researchers to present their work, get fruitful feedback from the discussion, and stimulate the interest of newcomers by giving them the opportunity to get a broad overview on the state-of-the-art in this area. This TSG refers to all levels of education

[1] Johannes Gutenberg University, Mainz, Germany. E-mail: yweiss@uni-mainz.de
[2] Amsterdam University of Applied Sciences, Amsterdam, the Netherlands.
E-mail: d.a.van.den.bogaart@hva.nl

— from primary school to tertiary education, including in-service teachers' training — preferably on work and conclusions based on actual classroom experiments and/or produced teaching and learning materials.

1.1 Submissions

We received 41 submissions from 16 countries (South America: 2; North America: 5; Asia: 23; Europe: 10; Africa: 1). Of those 41 submissions, 6 were rejected, 2 were redirected to another TSG, 6 were accepted as poster and 27 were accepted as paper presentations (long or short). The main part of the review process was organized and carried out by former TSG Chair Kathleen M. Clark, former Co-Chair Constantinos Tzanakis and former team member Uffe T. Jankvist.

Three papers were withdrawn by the authors after the postponement of the conference.

Of the remaining 24 accepted papers and 6 posters, only 13 papers and 2 posters were able to be presented during the conference. A list of the papers and authors are included in order of presentation and are organized in Tab. 1:

Tab. 1. List of papers presented

	Paper and author(s)
[1]	Methodological proposal for the analysis of historical sources of mathematics. ***Erika Zubillaga-Guerrero****, Flor Monserrat Rodríguez-Vásquez* (Mexico)*, and María Teresa González-Astudillo* (Spain).
[2]	Towards qualitative and participative research on history of mathematics in mathematics education: some arguments and possible paths. ***David Guillemette*** (Canada).
[3]	The application of HPM micro-video in the teaching of the binomial theorem. ***Jiaye Han*** (China).
[4]	The design and cases of primary school HPM micro-video. ***Zhuochen Li*** *and Jiaye Han* (China).
[5]	Combining cognitive demand with history of mathematics in mathematics (teacher) education. ***Desiree Agterberg*** (The Netherlands).
[6]	The Gradual Linearization of German Geometry Teaching. ***Ysette Weiss*** (Germany).
[7]	An empirical study on the impact of students' cognition through the concept of function teaching from the perspective of HPM in senior high school. ***Silu Liu*** *and Zhongyu Shen* (China).
[8]	Organum Mathematicum — a mathematical shrine as source for modern math education. ***Silvia Schöneburg-Lennert*** *and Thomas Krohn* (Germany).
[9]	The binary tree and its avatars: From Xiantian to the eternal symmetree. ***Jorge Soto-Andrade*** (Chile)*, Dandan Sun* (China)*, Daniela Diaz-Rojas* (UK)*, and Alexandra Yáñez-Aburto* (Chile).
[10]	An empirical research on the intension of mathematical culture based on the history of mathematics. ***Qing-chun Yu*** (China).
[11]	The development of teachers' MKT: a case study of HPM learning community. ***Zhongshu Shen*** *and Jiachen Zou* (China).
[12]	Enhancing mathematics teaching self-efficacy in pre-service teachers: effects of an HPM learning community in Shanghai. ***Haozhe Jiang*** (China).
[13]	A comparative study of the history of mathematics in high school mathematics textbooks in mainland China and Taiwan China. ***Peiyao Lei*** (China).

1.2 Sessions

A new team was formed to organize the hybrid sessions in which the papers were presented. The TSG Chair Ysette Weiss, Co-Chair Desiree Agterberg and team member Silvia Schöneburg-Lennert led the sessions. The sessions were attended by at least 21 participants. This number does not account for some of the live audience in the Shanghai conference room that were not on the list of authors nor paid participants for TSG-27.

The ICMI organizing committee granted our TSG four timeslots for presentations. Generally, all four sessions started with one or two 15 minutes *long* oral presentations and a discussion afterwards and one or two 10 minutes *short* oral presentations with collective discussion afterwards. At the end of our last session, we did a short group reflection.

2. Conference Themes

The thematic call for proposals for TSG-27 was broad and reflected main research areas in the history of mathematics in mathematics education:

- Theoretical and/or conceptual frameworks — in particular from general mathematics education research — for integrating history in mathematics education;
- History and epistemology implemented in mathematics education: Classroom experiments and teaching materials, considered from various perspectives; e.g., cognitive, didactical, pedagogical, affective, etc.;
- Surveys on the history of mathematics as it appears in curriculum and/or textbooks;
- Original sources in the classroom, and their educational effects;
- The role of history of mathematics in relation to the use of digital technologies in the teaching and learning of mathematics;
- History and epistemology as a tool for an interdisciplinary approach in the teaching and learning of mathematics and the sciences by unfolding their productive interrelations;
- Cultures and mathematics fruitfully interwoven.

Almost all presentations contributed not only to one of these subject areas, but affected several topics. The introduction to a conceptual frame for the inclusion of the history of mathematics and mathematics teaching in the teaching of mathematics was often accompanied by the discussion of its implementation, the display of empirical results and the consideration of possible educational effects of the use of historical sources as a tool.

Guillemette[2] reflected in his contribution on existing theoretical frameworks and their potentials and limits. Conceptual frameworks were also in the focus of the contribution by Zubillaga-Guerrero et al.[1]. They presented a methodology for the qualitative analysis of historical sources and demonstrated their tool in the analysis of

the concept of isomorphic groups in Arthur Cayley's work. Agterberg[5] introduced a cognitive demand framework that she developed for the analysis of tasks and classroom activities involving historical facts and sources and illustrated it with various examples. As the discussion showed, in several European and Asian countries there is an increasing involvement of history of mathematics in mathematics lessons and school curricula. Associated with this, one can find a growing number of contributions in mathematics textbooks that take the history of mathematics into account in varying ways.

Three contributions were directly devoted to curricular developments in textbooks, but in very different ways: in Weiss' study[6] of representations of conic sections in mathematics textbooks and curricula during the last 150 years, the focus was on the connections between institutional and curricular reforms. Lei's comparative textbook analysis[13] of high school mathematics textbooks in mainland China and Taiwan China concentrated on the integration of mathematics history. It was shown that the differences in the two editions were mostly related to applications. Yu[10] analysed 20 lessons from 2012–2018, regarding the implementation of the intention of mathematical culture in the reformed senior high school mathematics curriculum. All three contributions also examined the development of conceptual frameworks for the selection of curricular content.

Three papers with different emphases addressed the use of the history of mathematics to promote the understanding of mathematical concepts. Liu and Shen[7] studied the development of students' conceptual understanding of the concept of a function, using milestones of the historical development of this concept. Shen and Zou[11] presented a study on the development of teachers' mathematical knowledge over a semester at the HPM Studio in Shanghai.

The contributions by Li and Han[3,4] on the use of history-based micro videos (HPM micro-videos) in the teaching and learning of mathematics dealt with history of mathematics in relation to the use of digital technologies. The study focused on strengthening the motivation to learn mathematics and the selection of suitable topics for the promotion of general educational aspects. Jiang[12] presented a study, which also aimed on strengthening motivation, improvement of self-efficacy and changing beliefs, but this time in the context of an HPM teacher professional development program. The empirical study, he carried out, served him as a starting point for further conceptual development of this program.

In the contribution by Schöneburg-Lennert and Krohn[8], as well as in the contribution by Soto-Andrade et al.[9], the use of historical sources in mathematics lessons was foregrounded. The first contribution on the Organum Mathematicum presented teaching material from the 17th century and its educational potential. The second presentation of intercultural history of the concept development of the binary tree, beginning in ancient China in the 11th century up to the present, showed a variety of inspiring, enlightening and educational possibilities to approach mathematics and related subjects.

The great diversity of the presented historical sources made for an interesting and inspiring discussion. The wide range of implementations, which included primary and secondary education as well as university education and in-service teachers' training, also enabled a holistic view of mathematical-historical education.

3. Areas for Future Research

The topics discussed in the TSG-27 "The Role of the History of Mathematics in Mathematics Education" reflect actual research areas and significant recent scientific development in mathematics education.

The great importance of action and developmental research in our group raised the question of new formats with an international reach.

Another new accent was set by the question of possible cooperation between historians of mathematics, mathematicians, mathematics educators and mathematics teachers.

Since many topics that are relevant in other TSGs are also relevant to our TSG, the question of cooperation with other TSGs arose. A lot of TSGs come to mind, such as the teaching and learning of algebra/geometry/calculus, teacher education, mathematics and interdisciplinary education, the use of technology and so on. For example, digital media offer new opportunities to have historical sources more easily accessible in the classroom. This opens up new possibilities for accessing historical materials, for instance virtual museum tours or film material. Tools such as GeoGeobra can be used to rediscover Greek geometry or transcendental curves.